Leaving Certificate Maths – Paper 1 Mock

Name: Solutions

Section A	Concepts and Skills	100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

(a) The same number can be represented in different ways. Write two more representations of each of the numbers in the table below, by choosing only from the following list.

Note: some of the representations may be approximations.

-36, $\quad 9\%$, $\quad \frac{1}{9}$, $\quad \frac{1}{62}$, $\quad \frac{1}{81}$,

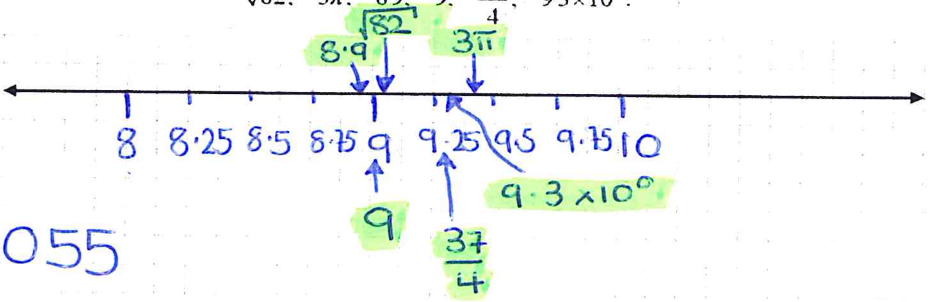
16^5, $\quad (\sqrt{3})^4$, $\quad 2^4 \times 2^5$, $\quad (0{\cdot}25)^{-10}$, $\quad 2{\cdot}8\%$,

36%, $\quad 900\%$, $\quad 0{\cdot}027$, $\quad 1{\cdot}05 \times 10^0$, $\quad 1{\cdot}05 \times 10^{-5}$.

6^{-2}	$0{\cdot}027$	$2{\cdot}8\%$
$81^{\frac{1}{2}}$	$(\sqrt{3})^4$	900%
2^{20}	$(0{\cdot}25)^{-10}$	16^5

(b) Giving the number line below an appropriate scale, mark the following numbers on it:

$\sqrt{82}$, $\quad 3\pi$, $\quad 8{\cdot}9$, $\quad 9$, $\quad \frac{37}{4}$, $\quad 9{\cdot}3 \times 10^0$.



$\sqrt{82} = 9{\cdot}055$

$3\pi = 9{\cdot}425$

$8{\cdot}9$

9

$\frac{37}{4} = 9{\cdot}25$

$9{\cdot}3 \times 10^0 = 9{\cdot}3$

page	running

Question 2 **(25 marks)**

(a) A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate (AER) of 6%. Find the value of the investment when it matures in eight years' time.

$F = P(1+i)^t$

$F = 5{,}000(1+0{\cdot}06)^8$

$F = 5{,}000(1{\cdot}06)^8$

$\boxed{F = €7{,}969{\cdot}24}$

(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

$F = P(1+i)^t$

$P = 5{,}000$

$i = ?$

$t = 8$ yrs

$F = 6{,}000$

$5{,}000 \times 20\% = 1{,}000$

$\therefore 5{,}000 + 1{,}000 = 6{,}000$ after 8 yrs.

$\Rightarrow 6{,}000 = 5{,}000(1+i)^8$

$\frac{6{,}000}{5{,}000} = (1+i)^8$

$1{\cdot}2 = (1+i)^8$

$\sqrt[8]{1{\cdot}2} = 1+i$

$1{\cdot}023 = 1+i$

$1{\cdot}023 - 1 = i$

$0{\cdot}023 = i \Rightarrow \boxed{i = 2{\cdot}3\%}$

* Complex Numbers *

Question 3 **(25 marks)**

(a) Write each of the following complex numbers in the form $a + bi$, where $i^2 = -1$.

$z_1 = (3+2i)(2-5i) =$ $(3+2i)(2-5i)$

$3(2-5i) + 2i(2-5i)$

$6 - 15i + 4i - 10i^2 = 6 - 11i - 10(-1)$

$= \boxed{16 - 11i}$

$z_2 = (5+4i)(17-13i) - (5+3i)(17-13i) =$

$i^2 = -1$

$(5+4i)(17-13i)$ | $(5+3i)(17-13i)$

$85 - 65i + 68i - 52i^2$ | $85 - 65i + 51i - 39i^2$

$85 + 52 + 3i = 137 + 3i$ | $85 + 39 - 14i = 124 - 14i$

$137 + 3i - (124 - 14i) = 137 + 3i - 124 + 14i$

$= \boxed{13 + 17i}$

$z_3 = \left(\frac{5}{2} + \frac{7}{2}i\right)^2 - \left(\frac{5}{2} + \frac{1}{2}i\right)^2 =$ $\left(\frac{5+7i}{2}\right)^2 - \left(\frac{5+1i}{2}\right)^2$

$= \frac{(5+7i)^2}{(2)^2} - \frac{(5+1i)^2}{(2)^2}$

$= \frac{25 + 70i + 49i^2}{4} - \frac{25 + 10i + i^2}{4} = \frac{-24 + 70i}{4} - \frac{24 + 10i}{4}$

$= \frac{1}{4}(-24 + 70i - 24 - 10i) = \frac{1}{4}(-48 + 60i) = \boxed{-12 + 15i}$

$z_4 = 1 + i + i^2 + i^3 =$

$= 1 + i + (-1) + (-1i)$

$= 1 + i - 1 - i$

$\boxed{= 0}$

(b) Which of z_1 and z_2 above is farther from 0 on an Argand diagram? Justify your answer. → hint to find the modulus!!!

$z_1 = 16 - 11i$

$|z_1| = \sqrt{(16)^2 + (-11)^2}$

$= \sqrt{377}$

$= 19.4$

$z_2 = 13 + 17i$

$|z_2| = \sqrt{(13)^2 + (17)^2}$

$= \sqrt{458}$

$= 21.4$

$\therefore$ z_2 is farther from 0 (the origin).

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Algebra

Question 4 (25 marks)

-b formula

(a) Solve the equation $x^2 - 6x - 23 = 0$, giving your answer in the form $a \pm b\sqrt{2}$, where $a, b \in \mathbb{Z}$.

$x^2 - 6x - 23 = 0$

$a = 1$
$b = -6$
$c = -23$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-23)}}{2(1)}$$

$$\frac{6 \pm \sqrt{36 + 92}}{2}$$

$$\frac{6 \pm \sqrt{128}}{2}$$ → type into calculator

$$\frac{6 \pm 8\sqrt{2}}{2}$$

$\Rightarrow \boxed{3 \pm 4\sqrt{2}}$

*nb!!

(b) Solve the simultaneous equations

$$2r - s = 10$$
$$rs - s^2 = 12$$

$2r - s = 10$

$-s = 10 - 2r$

Fill into quadratic equation → $\boxed{s = -10 + 2r}$

$rs - s^2 = 12$

$r(-10 + 2r) - (-10 + 2r)^2 = 12$

$-10r + 2r^2 - [(-10 + 2r)(-10 + 2r)] = 12$

$-10r + 2r^2 - [100 - 20r - 20r + 4r^2] = 12$

$-10r + 2r^2 - [100 - 40r + 4r^2] = 12$

$-10r + 2r^2 - 100 + 40r - 4r^2 = 12$

$-2r^2 + 30r - 100 - 12 = 0$

$-2r^2 + 30r - 112 = 0$

$(\div -2)$ $r^2 - 15r + 56 = 0$

$(r - 7)(r - 8) = 0$

$r - 7 = 0$ $r - 8 = 0$

$\boxed{r = 7}$ $\boxed{r = 8}$

$s = -10 + 2r$

$r = 7$
$s = -10 + 2(7)$
$\boxed{s = 4}$

$r = 8$
$s = -10 + 2$
$\boxed{s = 6}$

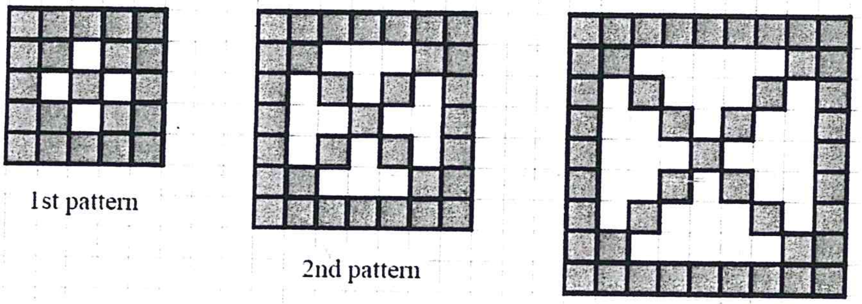
Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6.

Question 5 **(50 marks)**

Sile is investigating the number of square grey tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonals. There are no tiles in the white areas in the patterns – there are only the grey tiles.

(Questions start overleaf.)



1st pattern

2nd pattern

3rd pattern

answers over the page

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(a) In the table below, write the number of tiles needed for each of the first five patterns.

Pattern	1	2	3	4	5
No. of tiles	21	33	45	57	69

(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern.

from logs table → $\boxed{T_n = a + (n-1)d}$

first term difference.

$T_n = 21 + (n-1)12$

$= 21 + 12n - 12$

$\boxed{T_n = 12n + 9}$

(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.

$T_{10} = 12(10) + 9$

$= 129$ tiles

(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?

$T_n = 12n + 9$

* let $T_n = 399$ tiles *

$12n + 9 = 399$

$12n = 399 - 9$

$12n = 390$

$n = \frac{390}{12}$

$n = 32 \cdot 5$

$\therefore$ 32nd pattern is the biggest pattern Síle can make!

this means that it is a series!!!

(e) Find, in terms of n, a formula for the total number of tiles in the first n patterns.

$$S_n = \frac{n}{2}[\underbrace{2a}_{\text{first term}} + (n-1)\underbrace{d}_{\text{difference}}]$$

$$S_n = \frac{n}{2}[2(21) + (n-1)(12)]$$

$$= \frac{n}{2}[42 + 12n - 12]$$

$$= \frac{n}{2}[30 + 12n] = n[15 + 6n]$$

(f) Síle starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles, (21 + 33). How many patterns can she make in total with her 399 tiles?

$S_n = 399$, $S_n = n[15 + 6n]$

let $n[15 + 6n] = 399$

$$15n + 6n^2 = 399$$

$$6n^2 + 15n - 399 = 0$$

$(\div 3)$ $$2n^2 + 5n - 133 = 0$$

$$(2n + 19)(n - 7) = 0$$

$2n + 19 = 0$ | $n - 7 = 0$

$\cancel{n = -\frac{19}{2}}$ | $\boxed{n = 7}$

$\therefore$ she could make the first 7 patterns with her 399 tiles.

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Question 6 **(50 marks)**

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each day for the week, starting on the day that John got them.

Day	16cm high PLANT 1	24cm high PLANT 2
Day 1	16	24
Day 2	20	27·5
Day 3	24	31
Day 4	28	34·5
Day 5	32	38
Day 6	36	41·5
Day 7	40	45

*Note!! The 'Line' has come up on paper 1!!!

(b) Write down two formulas – one for each plant – to represent the plant's height on any given day. State clearly the meaning of any letters used in your formulas.

Let x = number of days
y = height of the plant (h)

* Both plants are growing at a constant rate ∴ they can both be graphed as straight lines *

Plant 1

$(1, 16)$ with x_1, y_1 — Day 1; $(7, 40)$ with x_2, y_2 — Day 7

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{40-16}{7-1} = \frac{24}{6} = 4$$

$$y - y_1 = m(x - x_1)$$
$$y - 16 = 4(x - 1)$$
$$y - 16 = 4x - 4$$
$$y = 4x + 12$$

h = 4(no. of days) + 12

Plant 2

$(1, 24)$ with x_1, y_1 — Day 1; $(7, 45)$ with x_2, y_2 — Day 7

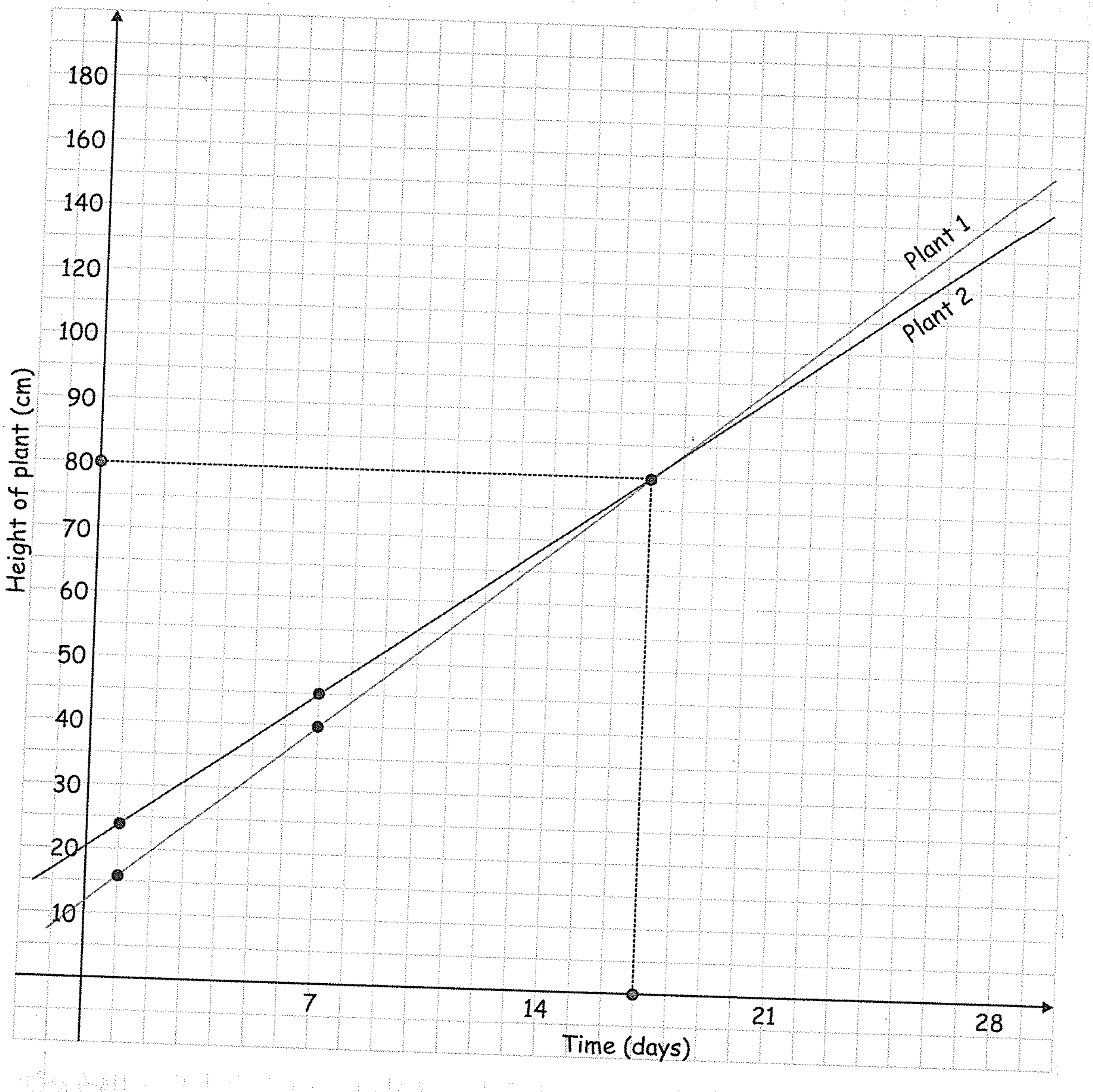
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{45-24}{7-1} = \frac{21}{6} = 3.5$$

$$y - y_1 = m(x - x_1)$$
$$y - 24 = 3.5(x - 1)$$
$$y - 24 = 3.5x - 3.5$$
$$y = 3.5x + 20.5$$

h = 3.5(no. of days) + 20.5

(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.

(Questions continue overleaf.)



(d) **(i)** From your diagram, write down the point of intersection of the two graphs.

Answer: (17, 80)

(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of *both* co-ordinates.

The point of intersection means that after 17 days both plants have the same height of 80cm.

(e) Check your answer to part **(d)(i)** using your formulae from part **(b)**.

$y = 4x + 12$

$\boxed{x = 17}$

$y = 4(17) + 12$

$y = 80$ ✓

$y = 3 \cdot 5x + 20 \cdot 5$

$\boxed{x = 17}$

$y = 3 \cdot 5(17) + 20 \cdot 5$

$y = 80$ ✓

(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

Algebra will give you the <u>exact</u> answer.

(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

Plants may not grow at the same rate per day - depending on weather. If they do not grow at the same rate then they will not be linear graphs.

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Question 7 * Functions & Calculus * **(50 marks)**

(a) Let $h(x) = x^2 + 1$, where $x \in \mathbb{R}$.

Write down a value of x for which $h(x) = 50$.

$h(x) = x^2 + 1$ $h(x) = 50$

$x^2 + 1 = 50$ $x^2 = 49$

$x^2 = 50 - 1$ $x = \sqrt{49}$

$x = 7$.

(b) Let $g(x) = \dfrac{1}{x-2}$, where $x \in \mathbb{R}$ and $x \neq 2$.

(i) Complete the following table:

x	0	1	1·5	1·75	2·25	2·5	3	4
$g(x)$	$-\frac{1}{2}$	−1	−2	−4	4	2	1	$\frac{1}{2}$

(ii) Draw the graph of the function g in the domain $0 \leq x \leq 4$.

x	$g(x) = \frac{1}{x-2}$	y	(x, y)
0	$\frac{1}{0-2} = -\frac{1}{2}$	$-\frac{1}{2}$	$(0, -\frac{1}{2})$
1	$\frac{1}{1-2} = -1$	-1	$(1, -1)$
2	$\frac{1}{2-2} = \frac{1}{0} = 0$	0	$(2, 0)$
3	$\frac{1}{3-2} = \frac{1}{1} = 1$	1	$(3, 1)$
4	$\frac{1}{4-2} = \frac{1}{2} = \frac{1}{2}$	$\frac{1}{2}$	$(4, \frac{1}{2})$

* Draw graph

Question 8 **(50 marks)**

(a) Differentiate $x^2 - 6x + 1$ with respect to x.

$$\frac{dy}{dx} = 2x - 6.$$

Speed / velocity
$\downarrow$ differentiate
Acceleration

(b) The speed, v, of an object at time t is given by

speed$\rightarrow$ $v = 96 + 40t - 4t^2$

where t is in seconds and v is in metres per second.

(i) At what times will the speed of the object be 96 metres per second?

let speed = 96 m/s.

$v = 96 + 40t - 4t^2$

$\cancel{96} = \cancel{96} + 40t \; 4t^2$

$-4t^2 + 40t = 0$

$(\div -4) \quad t^2 - 10t = 0$

$t(t-10) = 0$

$t = 0s$ or $t = 10s$.

(ii) What will the acceleration of the object be at $t = 2{\cdot}5$ seconds?

$v = 96 + 40t - 4t^2$

$\frac{dv}{dt} = 40 - 8t$

$\left.\frac{dv}{dt}\right|_{t=2\cdot5} = 40 - 8(2{\cdot}5)$

$= 20 \text{ m/s}^2$

* note!

m/s $\rightarrow$ speed

m/s^2 $\rightarrow$ acceleratio

(iii) At what value of t will the acceleration become $\boxed{\text{negative?}}$

$\frac{dv}{dt} < 0$

$40 - 8t < 0$

$-8t < -40$

$t > 5$.

* note
when you divide across an inequality by a negative number, the inequality turns around the opposite way.